Robust Online Hamiltonian Learning

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Motivation

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- Enabling adaptive measurement allows for large reductions in data collection costs.
- Want accurate reporting of errors incurred by estimate, and of smallest credible regions.

Learning Frequencies and T_2

Consider a single qubit undergoing Larmor precession at an unknown frequency ω:

$$H(\omega) = \frac{\omega}{2}\sigma_z, \quad |\psi_{\text{in}}\rangle = |+\rangle\,, \quad M = \{|+\rangle\langle+|\,, |-\rangle\langle-|\}$$

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If the qubit is also undergoing dephasing, we can model this by an exponentially decaying visibility, e^{-t/T_2}, where we would like to learn T_2.

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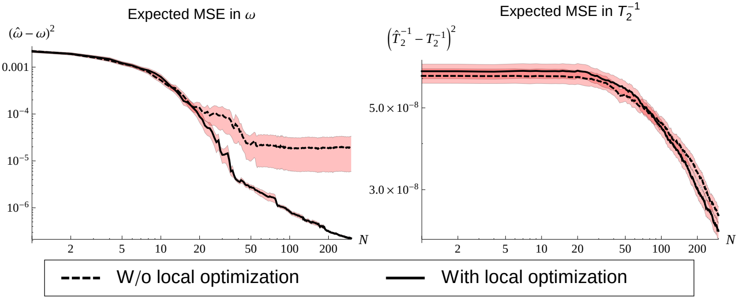

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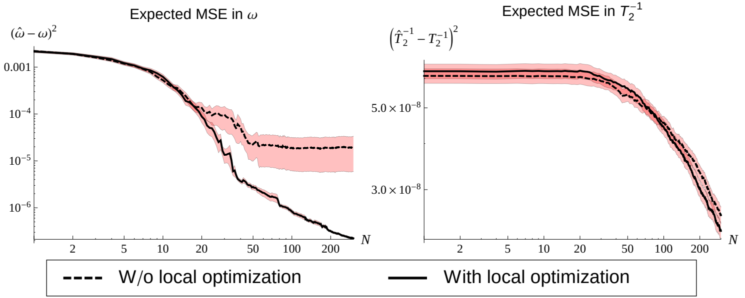

Moreover, using our best available knowledge to optimize experiment designs, we can continue to learn about ω even in the presence of unknown T_2.

Our Approach

Model data collection as a probability distribution $\Pr(D|\omega, T_2; t)$, and then update via Bayes' rule,

$$\Pr(\omega, T_2|D, t) = \frac{\Pr(D|\omega, T_2; t)}{\Pr(D|t)} \Pr(\omega, T_2),$$

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Simulation and learning are very closely connected.

Sequential Monte Carlo

To implement our approach on a computer, we approximate distributions by a sum over weighted delta functions,

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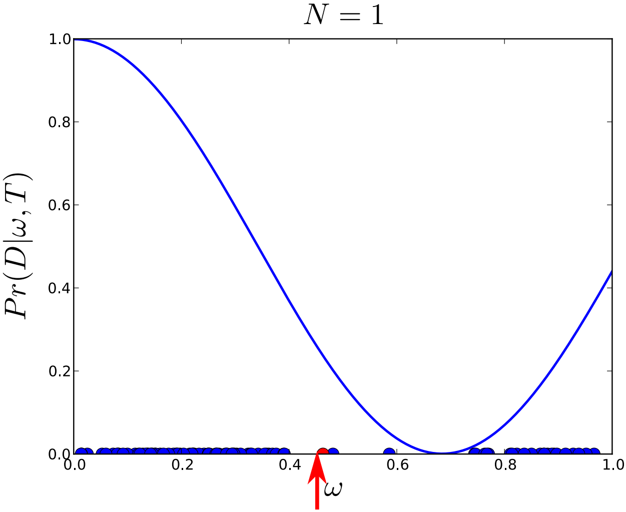
Updates to distributions now require evaluation the model $\Pr(D|\omega, T_2; t)$ at a finite number of points. Integrals over distributions are now represented by finite sums.

Sequential Monte Carlo

With SMC and resampling, particles move towards the true model as data is collected.



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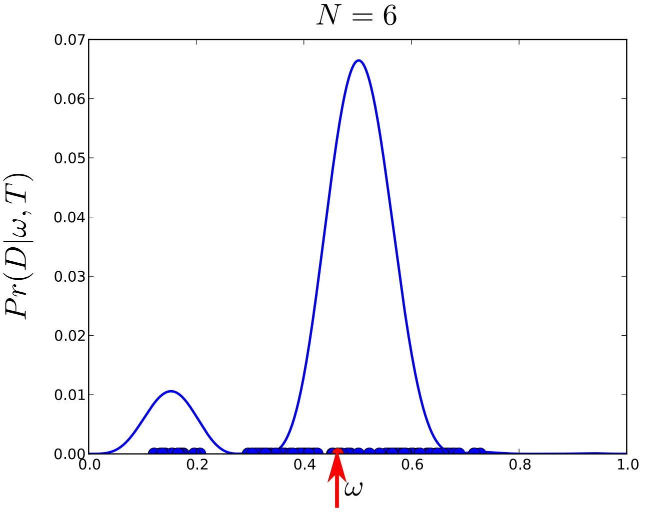

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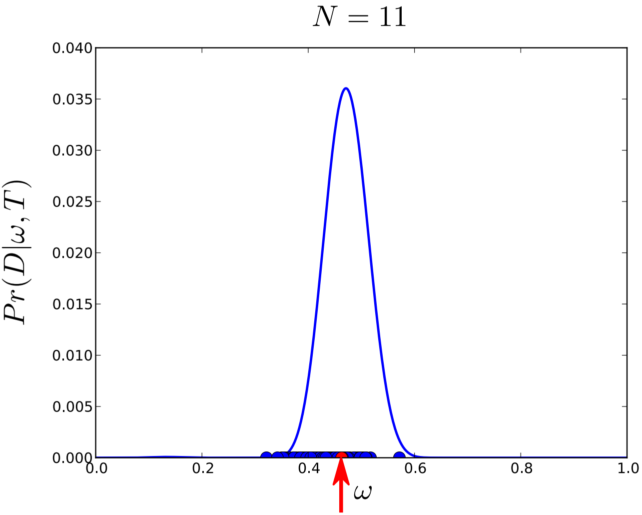

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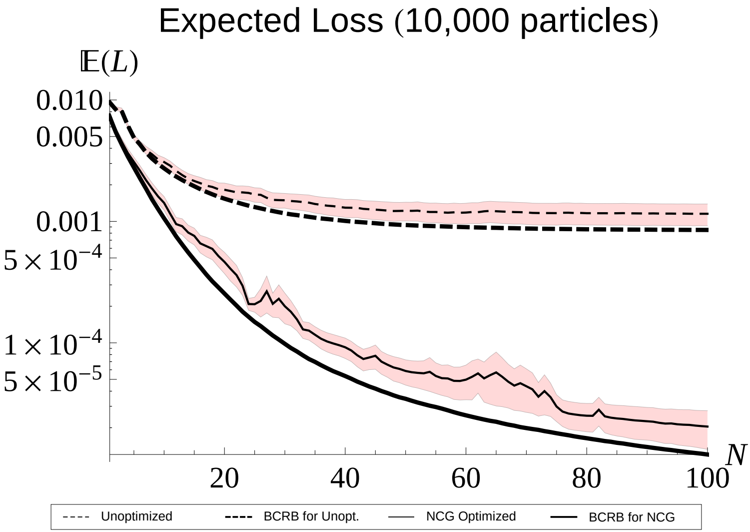
Because the sequential Monte Carlo approximation gives us a distribution, we can reason directly about the estimation error by performing a Bayes update for each possible measurement outcome, then finding the expected error given by the covariance.

Numerical Results: Adaptive Experiment Design



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- Sequential Monte Carlo allows for Bayesian updates to be efficiently implemented on a classical computer.
- Current best knowledge can be applied to adaptively design new experiments and measurements.
- Our approach is *generic*, treating simulation as a resource for learning.

Further Information

Slides, a journal reference for this work, a full bibliography and a software implementation can be found at *http://www.cgranade.com/research/rohl/*.

Thank you for your kind attention!