Robust Online Hamiltonian Learning

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Motivation

Accurate characterization of the dynamics of a quantum system is important for many applications in quantum information processing, including robust control design and simulation. Our goal here is to provide an algorithm to estimate parameters of unknown Hamiltonians, as well as quantify our uncertainty in those estimates. We do so by applying the methods of Bayesian inference and adaptive experiment design.

We benchmark our algorithm using several different models, the simplest being that of a single qubit undergoing Larmor precession at an unknown frequency $\omega$.

$$\Pr(\text{data} = 0 | \text{model} = \omega; \ t) = \cos^2(\omega t/2)$$

Here, $t$ is a control parameter that we can vary to adaptively query the dynamics of the model.

Sequential Monte Carlo Algorithm

We use the sequential Monte Carlo (SMC) algorithm to efficiently approximate expectation values over posterior distributions. This algorithm works by approximating the true distribution over models as a weighted sum of delta functions (often called “particles”):

$$\Pr(x|D, C) = \sum_i w_i \delta(x - x_i)$$

Expectation values can now be computed as finite sums, and the Bayes' Rule:

$$\Pr(\theta|D, C) = \frac{\Pr(D|\theta, C) \Pr(\theta|C)}{\int \Pr(D|\theta, C) \Pr(\theta|C) d\theta}$$

is then the expected negative quadratic loss averaged over the distribution of parameters. This provides an algorithm to estimate parameters of unknown Hamiltonians, including robust control design and simulation. Our goal here is to provide an algorithm to estimate parameters of unknown Hamiltonians, as well as quantify our uncertainty in those estimates. We do so by applying the methods of Bayesian inference and adaptive experiment design.

We infer parameters of a model by using Bayes' Rule:

$$\Pr(x|D, C) = \frac{\Pr(D|x, C)}{\Pr(D|C)} \Pr(x)$$

We judge the quality of an estimate by the squared error loss:

$$L(\hat{x}, x) = (x - \hat{x})^T (x - \hat{x})$$

Ultimately, the performance of any algorithm is measured by the quadratic loss averaged over the distribution of parameters. This quantity is called the Bayes risk:

$$r(\pi) = \mathbb{E}_{x, D|C}[L(\hat{x}, x)]$$

The utility of a hypothetical experiment C is then the expected negative of the risk function. Our algorithm adaptively designs experiments that efficiently provide information about the system by optimizing control parameters with respect to this utility.

Extension to Unknown $T_2$

We also consider models based on hyperparameters of some underlying distribution. For example, in the precession model, if the “true” $\omega$ is itself drawn from a Lorentz distribution with location $\omega_0$ and scale $\gamma$, then we obtain the unknown-$T_2$ model:

$$\Pr(0|\text{model} = (\omega_0, \gamma); t) = \frac{1}{2} + \frac{1}{2} e^{-t^2} \cos(\omega t)$$

Conclusions

Our work provides a simple algorithm that applies Bayesian inference to learn a Hamiltonian in an online fashion; that is to say, that our algorithm learns the Hamiltonian parameters as the experiment proceeds rather than collecting data and inferring the Hamiltonian through post-processing. This eliminates the need to store and process gigabytes of data that are recovered from even relatively short experiments. Our work has several advantages over existing approaches to learning Hamiltonian parameters. First, it can be used to estimate the optimal parameterization of the dynamics of an arbitrary quantum system within a space of model Hamiltonians. Second, it can be used to provide a region estimate of the Hamiltonian parameters. The importance of this is obvious: it allows us to not only learn the unknown parameters but also quantify our uncertainty in them. Third, our analysis of the algorithm shows a clear trade off between the experimental time and the computational time needed to parameterize the Hamiltonian.

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References


For more information and a software implementation, please visit http://www.cgranade.com/research/rohl or scan the code at left.